Study Guide – Rules for Transformations on a Coordinate Plane

Translations: one type of transformation where a geometric figure is "**slid**" horizontally, vertically, or both. Sliding a polygon to a new position without turning it. When translating a figure, every point of the original figure is moved the same distance and in the same direction.

Rules: A **positive** integer describes a translation **right or up** on a coordinate plane.
A **negative** integer describes a translation **left or down** on a coordinate plane.

(x, y) → (x ± a, y ± b)

*A movement **left or right** is on the x-axis. A movement **up or down** is on the y-axis.

Reflections: A type of transformation where a figure is "**flipped**" over a line of symmetry. A reflection produces a mirror image of a figure.

Rules: **Reflect a figure over the x-axis**: change the y-coordinates to their opposites.

(x, y) → (x, -y)

Reflect a figure over the y-axis: change the x-coordinates to their opposites.

(x, y) → (-x, y)

Reflect a figure over the y = x: change the coordinate positions

(x, y) → (y, x)

Reflect a figure over the y = -x: change the coordinate positions & signs

(x, y) → (-y, -x)

Rotations: A transformation that "**turns**" a figure about a fixed point given angle **degrees** and a **direction**.

Rules: **90°** clockwise rotation: **(x, y) → (y, -x)**
180° rotation: **(x, y) → (-x, -y)**
90° counterclockwise rotation: **(x, y) → (-y, x)**

Dilations: a transformation that **changes the size** of a figure, but not the shape.

Rule: To dilate a figure, always **MULTIPLY** the coordinates of each of its points by the factor of dilation.
Dilation factor **k**: **(x, y) → (kx, ky)**

Week 6: Transformations on a Coordinate Plane

Translations:

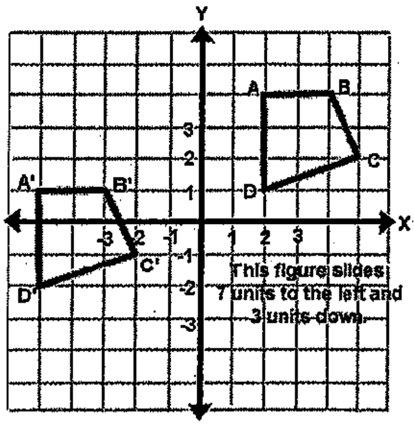
→ slides a figure horizontally, vertically, or both.
→ every point of the original figure is moved the same distance and same direction
→ The **image** (the new shape) will be exactly the same shape and same size as the original
- This means the original and the image are **congruent**

→ **Corresponding sides** in translated figures are **parallel.**

Rules: A **positive** integer describes a translation **right or up** on a coordinate plane.
A **negative** integer describes a translation **left or down** on a coordinate plane.
*A movement **left or right** is on the x-axis. A movement **up or down** is on the y-axis.

Translation of h, k:
$P(x, y) \rightarrow P'(x+h, y+k)$

If $h > 0$, the original point is shifted h units to the right
If $h < 0$, the original point is shifted to the left
If $k > 0$, the original graph is shifted k units up
If $k < 0$, the original point is shifted down.



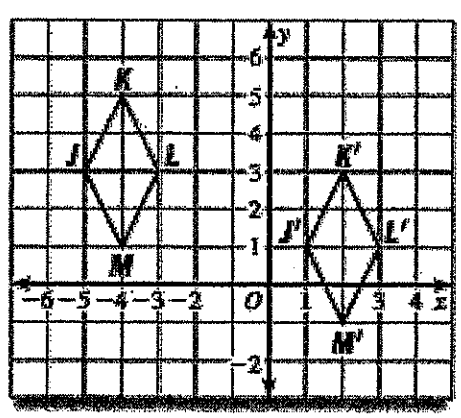
Example: For the translation shown in the graph, Quadrilateral ABCD was translated 7 units to the left and 3 units down to form its image A'B'C'D'
This can be written as $(x,y) \rightarrow (x-7, y-3)$
This may also be seen as $T_{-7,-3}(x,y) = (x-7, y-3)$

Example:

Draw quadrilateral *JKLM* with vertices $J(-5, 3)$, $K(-4, 5)$, $L(-3, 3)$, and $M(-4, 1)$. Then find the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 6, y - 2)$, and draw the image.

For each vertex of the original figure, add 6 to the *x*-coordinate and subtract 2 from the *y*-coordinate.

Original	**Image**
$J(-5, 3)$	$\rightarrow J'(1, 1)$
$K(-4, 5)$	$\rightarrow K'(2, 3)$
$L(-3, 3)$	$\rightarrow L'(3, 1)$
$M(-4, 1)$	$\rightarrow M'(2, -1)$



Each point on the original figure is translated 6 units to the right and 2 units down. The graph shows both figures.

Reflections

A reflection flips a figure across a line. That line is called the line of reflection.
→ The size and shape of the reflected figure does not change.
→ Each side and angle of a reflected figure corresponds to the same side and angle of its original
- A figure and its reflection are congruent.

Reflect a figure over the x-axis-

- when reflecting over the x-axis, change the y-coordinates to their opposites.
- (x,y) → **(x, -y)**

Reflect a figure over the y-axis-

- when reflecting over the y-axis, change the x-coordinates to their opposites.
- (x,y) → **(-x, y)**

Refect a figure over any other line –

- draw the line of reflection on the coordinate plane
- Count the distance of each point to the line of reflection.
- Plot the image of each point the same distance away from the line of reflection on the opposite side.

Example 1: Triangle ABC has vertices A(5,2), B(1,3), and C(-1,1). Find the coordinates of ABC after a reflection over the x-axis.

A(5,2) → (x, -y) → A'(5, -2)
B(1,3) → (x, -y) → B'(1, -3)
C(-1,1) → (x, -y) → C'(-1, -1)

Example 2: Quadrilateral KLMN has vertices K(2,3), L(5,1), M(4,-2), and N(1,-1). Find the coordinates of KLMN after a reflection over the y-axis.

K(2,3) → (-x, y) → K'(-2,3)
L(5,1) (-x, y) L'(-5, 1)
M(4,-2) (-x, y) M'(-4,-2)
N(1,-1) (-x, y) N'(-1,-1)

Draw parallelogram *ABCD* with vertices *A*(−3, 3), *B*(2, 3), *C*(4, 1), and *D*(−1, 1). Then find the coordinates of the vertices of the image after a reflection in the *x*-axis, and draw the image.

For each vertex of the original figure, multiply the *y*-coordinate by −1.

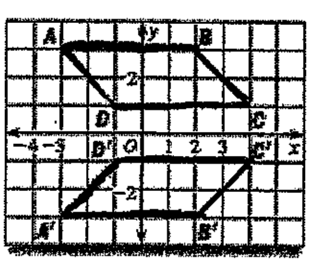
Original **Image**

A(−3, 3) → *A'*(−3, −3)

B(2, 3) → *B'*(2, −3)

C(4, 1) → *C'*(4, −1)

D(−1, 1) → *D'*(−1, −1)



The graph shows both figures.

Rotations:

- A transformation that **"turns"** a figure about a fixed point at a given angle and a given direction.

Rules:

→ **90 degree clockwise** rotation around the origin: (x, y) → **(y, -x)**

→ **90 degree counterclockwise** rotation **(x,y) → (- y , x)**

-- This is the same as a **270 degree** clockwise rotation around the origin: **(-y, x)**

→ **180 degree** rotation around the origin (0,0), use: → (x,y) → **(-x, -y)**

Example 1: Triangle NPQ has vertices N(0,0), P(4,-1), and Q(4,2). Rotate clockwise 90 degrees.

N(0,0)	(y, -x)	N'(0 ,0)
P(4,-1)	(y, -x)	P'(-1, -4)
Q(4,2)	(y, -x)	Q'(2, -4)

Example 2: Triangle KLM has vertices K(1,0), L(4,2), and M(3,4). Rotate 180 degrees.

K(1,0)	(-x, -y)	K'(-1,0)
L(4,2)	(-x, -y)	L'(-4,-2)
M(3,4)	(-x, -y)	M'(-3,-4)

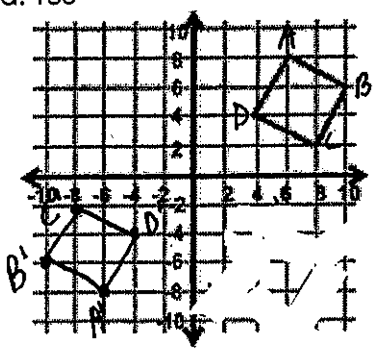
Example 3: Quadrilateral DEFG has vertices D(-1,0), E(-4,1), F(-3,3), and G(0,4). Rotate 90° counter clockwise.

D(-1,0)	(-y, x)	D'(0, -1)
E(-4,1)	(-y, x)	E'(-1, -4)
F(-3,3)	(-y, x)	F'(-3, -3)
G(0,4)	(-y, x)	G'(-4, 0)

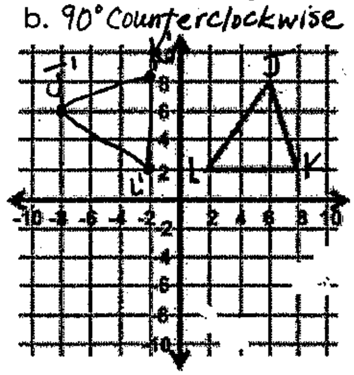
You Try:

1. **Rotate each figure about the origin.**

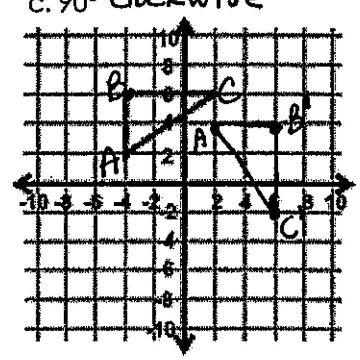
a. 180°



b. 90° Counterclockwise

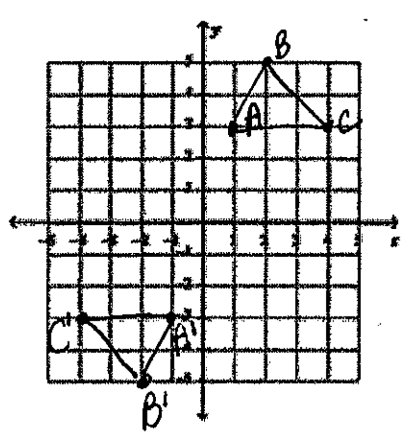


c. 90° clockwise



2. Draw a triangle with vertices A: (1,3), B: (2,5), and C:(4,3). Then determine the image of this triangle after performing a 180 degree rotation about the origin. Lable the image with A'B'C'.

A': (-1, -3)
B': (-2,5)
C': (-4, -3)



Dilations:

a transformation that **changes the size** of a figure, but not the shape.

→ can reduce or enlarge a shape.

→ each coordinate of a dilated figure is multiplied by a **scale factor, k.**

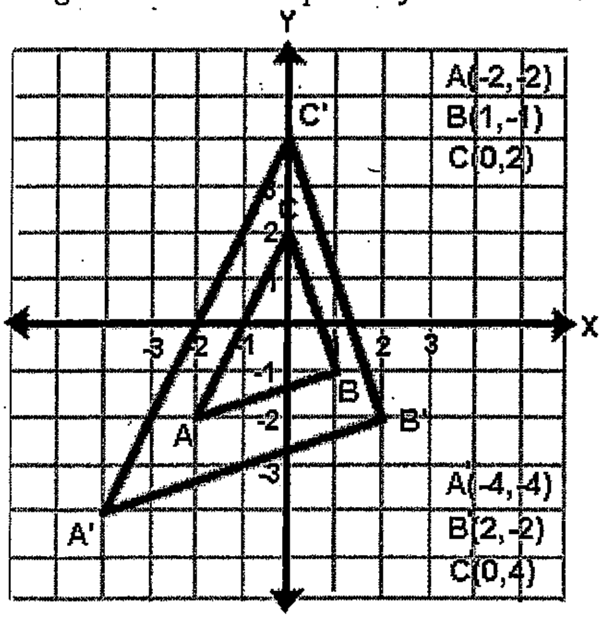
- if k is between 0 and 1, the figure is reduced
- if k is greater than 1m the figure is enlarged.

→ a dilated figure is **similar** (not congruent) to the original figure

- it has the same shape, but different size!
- Angle measures in similar figures are the same.
- The sides of similar figures are proportional.

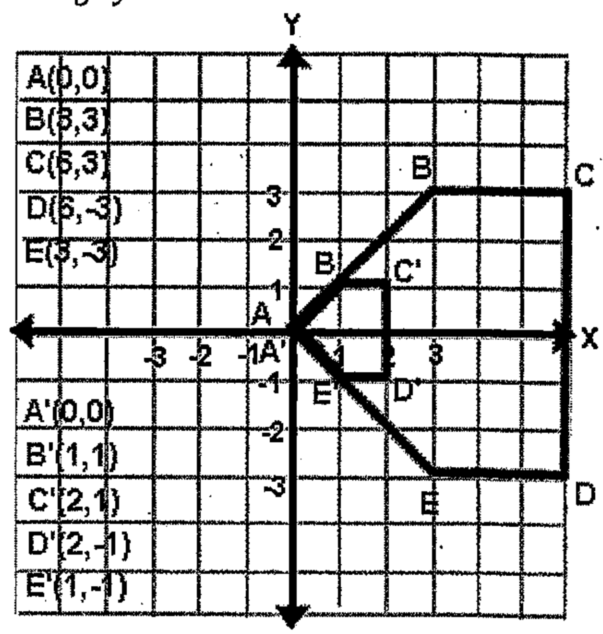
Example: Draw the dilation image of triangle ***ABC*** with the center of dilation at the origin and a scale factor of 2.

Notice how every coordinate of the original triangle has been multiplied by the scale factor 2.



Example: Draw the dilation image of pentagon ***ABCDE*** with the center of dilation at the origin and a scale factor of 1/3.

Notice how EVERY coordinate of the original pentagon has been multiplied by the scale factor (1/3). → *multiplying by 1/3 is the same as dividing by 3*



Sometimes the **center of dilation is not the origin.** When the origin is not the center of dilation, the distance from the center of dilation to each point on the original figure is multiplied by the scale factor.

Example: Draw the dilation image of rectangle ***EFGH*** with the center of dilation at point ***E*** and a scale factor of 1/2.

Notice: Point ***E*** and its image are the same. It is important to observe the **distance** from the center of the dilation, ***E,*** to the other points of the figure. Notice ***EF*** = 6 and ***E'F'*** = 3.